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ON A CONTACT PROBLEM FOR AN ELASTIC WEDGE*

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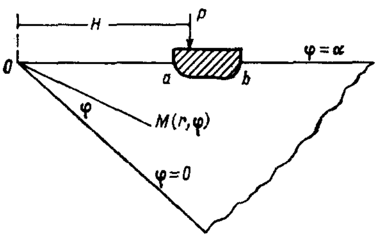
The problem of the impression of a stamp with arbitrary base in the face of an elastic wedge with the other face free is studied by an extension of the "small λ" asymptotic method /1/. Solutions are found specifically for an inclined and parabolic stamp. To confirm the accuracy of the asymptotic solutions, the problem is also investigated by a reduction to two infinite algebraic systems of equations of the second kind. The results can be utilized to refine the procedure for computing the contact strength of gear transmission. Only the case of a stamp with a flat base, which is of the least interest for applications, has been studied previously /2, 3/.

1. We examine the problem of the plane attain of an elastic wedge of aperture α on whose upper face a rigid stamp is impressed by a force P (see the figure). The arm of application of the force relative to the wedge apex is H, there are no friction forces between the stamp and wedge surfaces, and the wedge lower face is not loaded.



$$\sigma_\varphi(r, 0) = \tau_{r\varphi}(r, 0) = 0 \quad (0 < r < \infty) \tag{1.1}$$
$$\tau_{r\varphi}(r, \alpha) = 0 \quad (0 < r < \infty),$$
$$\sigma_\varphi(r, \alpha) = 0 \quad (0 < r < a, \ b < r < \infty)$$
$$v(r, \alpha) = -(\delta + \beta r - f(r)) = -\delta(r) \quad (a \leqslant r \leqslant b)$$

Here $\delta + \beta r$ is the rigid displacement of the stamp. The contact zone is governed by the inequality $a \leqslant r \leqslant b$, while the shape of the stamp base is the function $f(r)$. It is required to determine $\sigma_\varphi(r, \alpha) = -q(r)$ for $a \leqslant r \leqslant b$ as well as the connection between the quantities PH and β.

An integral equation of the problem has been obtained /2/ for the dimensionless function of the contact pressure $\varphi(x)$

$$\frac{1}{\lambda}\int_{-1}^{1}\varphi(\xi)K\left(\frac{\xi - x}{\lambda}\right)d\xi = \pi g(x) \quad (|x| \leqslant 1) \tag{1.2}$$

$$K(t) = \int_0^\infty L(u, \alpha)\sin ut\, du - \frac{\pi B(\alpha)}{2} \tag{1.3}$$

$$L(u, \alpha) = (u\sin 2\alpha + \operatorname{sh} 2\alpha u)(2\operatorname{sh}^2\alpha u - 2u^2\sin^2\alpha)^{-1} \tag{1.4}$$

Here

$$\lambda = 2[\ln(b/a)]^{-1}, \ x = \lambda\ln(r/a) - 1, \ \varphi(x) = rq(r) \tag{1.5}$$
$$g(x) = \theta r\delta'(r), \ \theta = G(1 - \nu)^{-1}$$

(G is the shear modulus, and ν is Poisson's ratio). A table of values of $B(\alpha)$ is presented in /2/. The dimensionless parameter λ characterizes the relative location of the stamp on the wedge face.

For a plane, taking the slant into account $(\beta \neq 0)$, i.e., on inclined stamp

$$f(r) \equiv 0, \ g(x) = ke^{(x+1)/\lambda}, \ k = \beta a\theta \tag{1.6}$$

For a parabolic stamp (R is the radius of curvature)

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$$f(r) = r^2/(2R) - r(a+b)/(2R) + (a+b)^2/(8R) \tag{1.7}$$

$$g(x) = me^{(x+1)/\lambda} - ne^{2(x+1)/\lambda}$$

$$m = \theta a(\beta + (a+b)/(2R)),\quad n = \theta a^2/R$$

The following expressions are found by the method of "large λ." /1, 2/ for the inclined stamp (1.8) and the parabolic stamp (1.9):

$$\begin{aligned} q(r) = q_0\Big(&\frac{P}{\pi} + \mu\frac{PA}{2} - \mu^2\frac{Pa_0}{\pi} + \mu^3\frac{k}{2} - \mu^4\frac{Pa_1}{\pi} + \mu^5\frac{k}{24} + \\ &\lambda^{-1}\Big(\mu^2 k + \mu^4\frac{k}{6}\Big) + \lambda^{-2}\Big(\frac{Pa_0}{2\pi} + \mu\frac{PAa_0+k}{4} - \mu^2\frac{Pa_1}{\pi} + \\ &\mu^3\Big(\frac{3}{4}PAa_1 + \frac{11k}{48}\Big)\Big) + \lambda^{-3}\Big(\mu^2\frac{k}{12} - \frac{k}{2}\Big) + \lambda^{-4}\Big(\frac{7Pa_1}{8\pi} + \\ &\mu\Big(\frac{1}{8}PAa_0^2 - \frac{3}{16}PAa_1 - \frac{17k}{192} + \frac{5a_0k}{16}\Big)\Big) - \lambda^{-5}\frac{5k}{48}\Big) + O(\lambda^{-6}) \end{aligned} \tag{1.8}$$

$$\begin{aligned} q(r) = q_0\Big(&\frac{P}{\pi} + \mu\frac{PW}{2} - \mu^2\frac{Pa_0}{\pi} + \mu^3\Big(\frac{m}{2} - 2n\Big) - \\ &\mu^4\frac{Pa_1}{\pi} + \mu^5\Big(\frac{m}{24} - \frac{2n}{3}\Big) + \lambda^{-1}\Big(\mu^2(m-4n) + \\ &\mu^4\Big(\frac{m}{6} - \frac{8n}{3}\Big)\Big) + \lambda^{-2}\Big(\frac{Pa_0}{2\pi} + \mu\Big(\frac{PWa_0+m}{4} - n\Big) - \\ &\mu^2\frac{Pa_1}{\pi} + \mu^3\Big(\frac{3}{4}PWa_1 + \frac{11m}{48} - \frac{11n}{3}\Big)\Big) + \lambda^{-3}\Big(\frac{5n}{4} - \frac{m}{2} + \\ &\mu^2\Big(\frac{m}{12} - \frac{4n}{3} + \frac{3na_0}{4}\Big)\Big) + \lambda^{-4}\Big(\frac{7Pa_1}{8\pi} + \mu\Big(\frac{1}{8}PWa_0^2 - \\ &\frac{3}{16}PWa_1 - \frac{17m}{192} - \frac{223n}{192} + \frac{5a_0m}{16} + \frac{7a_0n}{16}\Big)\Big) + \\ &\lambda^{-5}\Big(\frac{95n}{48} - \frac{5m}{48} - \frac{3a_0n}{8}\Big)\Big) + O(\lambda^{-6}) \end{aligned} \tag{1.9}$$

$$q_0 = q_0(r) = \Big(r\sqrt{\ln\frac{b}{r}\ln\frac{r}{a}}\Big)^{-1},\quad \mu = \mu(r) = \ln\frac{r}{\sqrt{ab}}$$

$$A = B + \frac{2k}{P},\quad W = B + 2\frac{m-n}{P}$$

It can be shown that for $\beta = 0$ solution (1.8) agrees with (3.13) in /2/ while (1.9) agrees with (1.8) for $R = \infty$.

Values of $a_0(\alpha)$ and $a_1(\alpha)$ are given below for the practical utilization of (1.8) and (1.9):

$12\alpha/\pi$	1	2	4	6	8	10	18
$-a_0$	498	56.1	4.98	0.876	0.207	$9.29\cdot10^{-2}$	$4.22\cdot10^{-2}$
a_1	667	34,7	1.30	0.122	0.0134	$2.01\cdot10^{-3}$	$0,399\cdot10^{-3}$

2. Let us describe the modified method of "small λ" /1/. We separate (1.2) and (1.3) into a system of two integral equations

$$\begin{aligned} &-\int_{-1}^{\infty}\varphi_1(\xi)\Big(l\Big(\frac{x-\xi}{\lambda}\Big) + \frac{\pi B}{2}\Big)d\xi = \\ &\quad \pi\lambda g_1(x) - \int_{-\infty}^{-1}\varphi_2(\xi)\Big(l\Big(\frac{x-\xi}{\lambda}\Big) + \frac{\pi B}{2}\Big)d\xi \quad (-1 \leqslant x < \infty) \\ &\int_{-\infty}^{1}\varphi_2(\xi)\Big(l\Big(\frac{x-\xi}{\lambda}\Big) + \frac{\pi B}{2}\Big)d\xi = \\ &\quad -\pi\lambda g_2(x) - \int_{1}^{\infty}\varphi_1(\xi)\Big(l\Big(\frac{x-\xi}{\lambda}\Big) + \frac{\pi B}{2}\Big)d\xi \quad (-\infty < x \leqslant 1) \end{aligned} \tag{2.1}$$

Here

$$l(t) = \int_0^{\infty} L(u, \alpha)\sin ut\, du$$

$$g(x) = g_1(x) - g_2(x),\quad \varphi(x) = \varphi_1(x) - \varphi_2(x) \quad (|x| \leqslant 1)$$

We assume that

$$g_1(x) = O(e^{-\alpha_1 x}),\quad \varphi_1(x) = O(e^{-\beta_1 x}),\quad x \to +\infty$$

$$g_2(x) = O(e^{\alpha_2 x}),\quad \varphi_2(x) = O(e^{\beta_2 x}),\quad x \to -\infty$$

where $\alpha_1, \alpha_2, \beta_1, \beta_2$ are positive constants. We introduce the notation

$$\varphi_1(x)=\psi_+\left(\frac{1+x}{\lambda}\right),\quad \varphi_2(x)=\psi_-\left(\frac{1-x}{\lambda}\right)$$
$$g_1(x)=y_+\left(\frac{1+x}{\lambda}\right),\quad g_2(x)=y_-\left(\frac{1-x}{\lambda}\right)$$

After obvious changes of variables in (2.1), we arrive at the system

$$-\int_0^\infty \psi_\pm(\tau)\left(l(t-\tau)\pm\frac{\pi B}{2}\right)d\tau= \tag{2.2}$$
$$\pi y_\pm(t)-\int_{2/\lambda}^\infty \psi_\mp(\tau)\left(l\left(t+\tau-\frac{2}{\lambda}\right)\pm\frac{\pi B}{2}\right)d\tau\quad (0\leqslant t<\infty)$$

It can be shown that as $t\to\infty$ the function $l(t)$ tends to $(2\pi B)^{-1}\operatorname{sgn} t$ and the next term in the asymptotic representation decreases exponentially. If $t\to 0$ then $l(t)=O(t^{-1})$. Taking account of the above and the asymptotic behaviour of the functions $\psi_\pm(t)$ at infinity, it can be asserted that the integrals on the right-hand sides of (2.2) are exponentially small components for small λ and $t>0$. Consequently, the principal term of the asymptotic form of the solution of the problem for small λ can be constructed by solving a Wiener-Hopf integral equation /4/ of the form

$$-\int_0^\infty \psi(\tau)\left(l(t-\tau)\pm\frac{\pi B}{2}\right)d\tau=\pi e^{-\varepsilon t}\quad (0\leqslant t<\infty) \tag{2.3}$$

Such solutions are obtained below in approximating the function $L(u,\alpha)$, taking its properties into acoount /2/, by the expression $(u^2+C^2)(u+\sqrt{u^2+D^2})^{-1}$, where ε is a complex constant and C and D are real constants.

For the plus sign in the kernel

$$\psi(t)=-z_1(t)\frac{\sqrt{D+\varepsilon}}{C+\varepsilon}+\varepsilon\frac{\sqrt{D^2-\varepsilon^2}}{C^2-\varepsilon^2}z_2(t,\varepsilon) \tag{2.4}$$
$$(-\min(D,C)<\operatorname{Re}\varepsilon)$$
$$z_1(t)=e^{-Dt}/\sqrt{\pi t}+\sqrt{D-C}\,e^{-Ct}\operatorname{erf}\sqrt{(D-C)t} \tag{2.5}$$
$$z_2(t,\varepsilon)=e^{-\varepsilon t}\operatorname{erf}\sqrt{(D-\varepsilon)t}-$$
$$(\sqrt{D-C}/\sqrt{D-\varepsilon})\,e^{-Ct}\operatorname{erf}\sqrt{(D-C)t}$$

For the minus sign

$$\psi(t)=\varkappa z_1(t)+\varepsilon\sqrt{D^2-\varepsilon^2}\,z_2(t,\varepsilon)\quad(0<\operatorname{Re}\varepsilon) \tag{2.6}$$

Here $\varkappa$ is an arbitrary constant. Formulas (2.4) and (2.6) are easily rewritten for the cases $\varepsilon=C, \operatorname{Re}\varepsilon>D$ or $C>D$.

It can be shown /1/ that the components of the asymptotic form of the principal term are exponentially small for small λ and, consequently, are not essential for the calculation.

3. The representations

$$\operatorname{cth}\frac{u}{B}=\frac{B}{u}\prod_{n=1}^\infty\left(1+\frac{u^2}{\delta_n^2}\right)\left(1+\frac{u^2}{\gamma_n^2}\right)^{-1},\quad \gamma_n=B\pi n, \tag{3.1}$$
$$\delta_n=B\pi\left(n-\frac{1}{2}\right)$$
$$\operatorname{cth}\frac{u}{B}=\frac{B}{u}+\sum_{n=1}^\infty\frac{2Bu}{u^2+\gamma_n^2} \tag{3.2}$$

hold /5, 6/.

The **meromorphic** function **cth** $B^{-1}u$ has the zeros $\zeta_n=i\delta_n$ and poles $z_n=i\gamma_n\ (n=1,2\ldots)$ in the half-plane $\operatorname{Im}u>0$ of the complex variable u. The identities

$$\frac{1}{\zeta_m}-\sum_{n=1}^\infty\frac{2\zeta_m}{z_n^2-\zeta_m^2}=0\quad (m=1,2\ldots) \tag{3.3}$$

result from (3.2).

The function $L(u, \alpha)$ can be approximated by $\operatorname{cth} B^{-1}u$ with comparatively small error /2, 3/. For such an approximation the solution of (2.3) for $\varepsilon = -i\eta$ $(\operatorname{Im}\eta > 0)$ can be represented for the plus sign as follows:

$$\psi(t) = -\frac{ie^{i\eta t}}{\operatorname{cth} B^{-1}\eta} - i\sum_{n=1}^{\infty} C_n(\eta)\, e^{i\zeta_n t} \tag{3.4}$$

$$C_n(\eta) = \zeta_n\,(BN(\eta)N'(-\zeta_n)(\eta-\zeta_n))^{-1}$$

$$N(u) = \prod_{n=1}^{\infty}\left(1+\frac{u}{\zeta_n}\right)\left(1+\frac{u}{z_n}\right)^{-1}$$

For the right-hand side of (2.3) equal to $\pi e^{iz_m t}$ we obtain

$$\psi(t) = -i\sum_{n=1}^{\infty} C_n(z_m)\, e^{i\zeta_n t} \tag{3.5}$$

For the minus sign the solution of (2.3) for $\varepsilon = -i\eta$ has the form (3.4) with the additional component $\varkappa \sum_{n=1}^{\infty} e^{i\zeta_n t}/N'(-\zeta_n)$, ($\varkappa$ is an arbitrary constant) corresponding to the solution of the homogeneous integral equation.

Let us substitute (3.4) into (2.3) with the plus sign (everything is done analogously for the minus sign), whose kernel is represented in the form of the series /5/

$$\int_0^{\infty} \operatorname{cth} B^{-1}u \sin(x-t)\,u\,du \pm \frac{\pi B}{2} = \tag{3.6}$$

$$\frac{\pi B}{2}(\operatorname{sgn}(x-t)\pm 1) + \pi B \operatorname{sgn}(x-t)\sum_{n=1}^{\infty} e^{-|x-t|\gamma_n}$$

Using the identity (3.3) and the independence of the system of functions $\{e^{ixz_n}, n = 1, 2\ldots\}$ /1/, we arrive at an infinite system of linear algebraic equations

$$\frac{1}{\operatorname{cth} B^{-1}\eta\,(z_m-\eta)} + \sum_{n=1}^{\infty}\frac{C_n(\eta)}{z_m-\zeta} = 0 \quad (m = 1, 2\ldots) \tag{3.7}$$

If $\pi e^{iz_m t}$ is on the right-hand side, we obtain analogously

$$\sum_{n=1}^{\infty}\frac{BN(z_m)\,C_n(z_m)}{\zeta_n\,[N^{-1}(-z_m)]'\,(z_j-\zeta_n)} = \delta_{mj} \quad (m, j = 1, 2\ldots) \tag{3.8}$$

where δ_{mj} is the Kronecker delta. As follows from (3.8), the matrix $\{BC_n(zm)N(z_m)(\zeta_n [N^{-1}(-z_m)]')^{-1};\ n, m = 1, 2\ldots\}$ is the right-sided inverse to the matrix $\{(z_j-\zeta_n)^{-1};\ j, n = 1, 2\ldots\}$. It can be proved /1/ that it is also the left-sided inverse. Therefore, system (3.7) with the singular matrices can be solved effectively.

The $L(u, \alpha)$ in (1.4) is a function of the cth type and representable in the forms of (3.1) and (3.2). Its zeros and poles are roots of the transcendental equations

$$\operatorname{sh} u \pm \alpha^{-1}\sin\alpha u = 0$$

Several of the first roots are presented for certain α in /7/. The asymptotic form of large zeros and poles (numbering as on p.216 in /1/), respectively, has the form

$$\zeta_{2n+1}^{*} \sim \left(\frac{1}{2\alpha}\ln n + \frac{1}{2\alpha}\ln\frac{4\pi|\sin 2\alpha|}{\alpha}\right) + i\left(\frac{2\pi}{\alpha}n + \frac{3\pi}{4\alpha}\right) \tag{3.9}$$

$$z_{2n+1}^{*} \sim \left(\frac{1}{\alpha}\ln n + \frac{1}{\alpha}\ln\frac{8\pi|\sin\alpha|}{\alpha}\right) + i\left(\frac{4\pi}{\alpha}n + \frac{3\pi}{2\alpha}\right)$$

We henceforth omit the asterisk. Now, the results of (3.3)-(3.8) can be applied to (2.2) with functions $L(u, \alpha)$ of the form (1.4).

In the case of an inclined stamp $y_+(t) \equiv 0$, $y_-(t) = -ke^{-t+2/\lambda}$. Consequently, we seek the solution of system (2.2) in the form

$$\psi_+(t) = i\sum_{n=1}^{\infty} E_n e^{i\zeta_n t} - iS\sum_{n=1}^{\infty} e^{i\zeta_n t}/N'(-\zeta_n) \tag{3.10}$$

$$\psi_-(t) = -\frac{ike^{-t+2/\lambda}}{BN(i)\,N(-i)} + i\sum_{n=1}^{\infty} D_n e^{i\zeta_n t} - i\varkappa \sum_{n=1}^{\infty} \frac{e^{i\zeta_n t}}{N'(-\zeta_n)}$$

$$\sum_{n=1}^{\infty}\frac{E_n}{\zeta_n} - \sum_{n=1}^{\infty}\frac{D_n e^{i\zeta_n 2/\lambda}}{\zeta_n} - \frac{k}{BN(i)\,N(-i)} + \tag{3.11}$$

$$S\sum_{n=1}^{\infty}\frac{1}{N'(-\zeta_n)\zeta_n} + \varkappa\sum_{n=1}^{\infty}\frac{e^{i\zeta_n 2/\lambda}}{N'(-\zeta_n)\,\zeta_n} = 0$$

Substituting (3.10) and (3.11) into (2.2) and using the representation

$$l\left(t+\tau-\frac{2}{\lambda}\right) \pm \frac{\pi B}{2} = \pi B\left(\frac{1}{2} \pm \frac{1}{2}\right) + \pi B \sum_{n=1}^{\infty} e^{i(t+\tau-2/\lambda)z_n}$$

and then inverting the singular matrix, as shown above, we arrive at two infinite systems of linear algebraic equations in the new unknowns $x_n^{\pm} = D_n \pm E_n$:

$$X_{\pm} = \mp F X_{\pm} + Q_{\pm} \tag{3.12}$$

where $X_{\pm} = \{x_n^{\pm}\}$, $Q_{\pm} = \{q_n^{\pm}\}$ $(n = 1, 2 \ldots)$ are columns and $F = \{f_{nm}\}$ $(n,\ m = 1, 2 \ldots)$ is a matrix, where

$$f_{nm} = \zeta_m N(\zeta_m)\, e^{i\zeta_m 2/\lambda} (\zeta_n N'(-\zeta_n)(\zeta_n + \zeta_m))^{-1}$$

$$q_n^{\pm} = (S+\varkappa)\sum_{n=1}^{\infty}\frac{f_{nm}}{N'(-\zeta_m)} \mp \frac{S}{N'(-\zeta_n)} -$$

$$\frac{ik}{BN(i)\,N(-i)\,N'(-\zeta_n)} \sum_{m=1}^{\infty} \frac{(z_m+i)\,e^{2/\lambda} \pm (z_m - i)}{[N^{-1}(-z_n)]'\,(z_m-\zeta_n)(z_m^2+1)}$$

System (3.12) are solved by using the additional relationship (3.11) to determine S. It can be proved that for these systems the method of successive approximations is applicable. An analogous pair of systems is obtained in the case of a parabolic stamp.

		Inclined Stamp		
1	0,484	1,687	1,718	0,926
	0,493	1,695	1,711	0,944
	0,492	1,681	1,718	0,921
	0,294	3,509	0,808	0,821
2	0,288	3,533	0,793	0,813
	0,281	3,514	0,798	0,815
		Parabolic stamp		
1	0,401	1,943	1,236	0,401
	0,408	1,937	1,219	0,408
	0,413	1,935	1,223	0,413
2	0,137	6,121	0,199	0,137
	0,146	6,089	0,202	0,146
	0,129	6,103	0,213	0,129

Bounded solutions /8/ on one end of the contact domain $(r = b)$ for the inclined stamp and on both ends for the parabolic stamp were used to calculate different characteristics of the problem by the "large λ" and "small λ" methods. The stamp equilibrium condition enables us to express $\varkappa$ in terms of P. In the "small λ" method, the integral on the right-hand side of (2.2) with the lower sign is initially neglected to find the solution and then $\psi_-(t)$ and $\psi_+(t)$ are determined successively from (2.2) with the upper sign. As estimates show, $\lambda > \alpha^{-1}$ can be considered to be large.

The quantities

$$q(\sqrt{ab})\,\frac{a}{P},\quad \frac{1}{aP}\int_a^b q(r)\,r\,dr = \frac{H}{a}$$

are respectively presented in the first two columns of the table.

The respective values

$$-\frac{k}{P},\quad \lim\left\{q(r)\,\frac{a}{P}\ln\sqrt{\frac{r}{a}}\right\}\quad (r\to a)$$

are given in the third and fourth columns for the inclined, and mP^{-1} and nP^{-1} for the parabolic stamp. All the quantities are computed by the three methods described above. The number 1 on the left corresponds to the case $\alpha = 0.5\pi, \lambda = 2$ and the number 2 to $\alpha = 1.5\pi, \lambda = 1$. The values of the constants C and D are taken from /2/.

Therefore, as in /9/, devoted to a wedge with a clamped lower face, joining of the asymptotic solutions for large and small λ has successfully been established for a wedge lower face is stress-free. The method of reducing the integral equation with a symbol of the type cth to infinite systems of the second kind that enable the accuracy of the asymptotic solutions to be monitored can also be considered effective.

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ASYMPTOTIC INTEGRATION OF NON-LINEAR EQUATIONS OF CYLINDRICAL PANEL VIBRATIONS*

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Complete asymptotic expansions of the solution of the two-dimensional problem of the non-linear vibrations of a cylindrical panel with free curvilinear boundaries are constructed using the boundary layer method /1, 2/ in the case when the parameter δ, equal to the ratio between the lengths of the clamped and free sides, is fairly small. The principal term of the **expansion** for the deflection function is determined from the known non-linear integrodifferential equation of arch vibrations. The discrepancies in satisfying the boundary conditions on the clamped boundaries turn out to be higher-order infinitesimals in δ and are compensated by boundary layer functions that are determined from linear boundary value problems for a biharmonic operator in a half-strip. Calculations are performed by using finite differences for elastic, elasto-plastic cylindrical panels, arches, and rectangular plates subjected to an instantaneously applied transverse step load, and the limits of applicability are established for a monomial expansion. Questions on passage

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